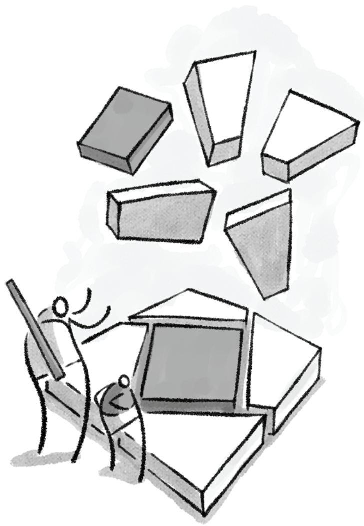
Making every maths lesson count



Six principles to support great maths teaching

Emma McCrea

Edited by Shaun Allison and Andy Tharby

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Foreword by Dylan Wiliam

There is probably no school subject in which public perceptions of the subject are more at variance with the views of its practitioners than in mathematics. I sometimes think that non-mathematicians imagine that mathematicians spend most of their time doing complex long division. Many years ago, in a book entitled *Do You Panic About Maths?*, Laurie Buxton, a mathematics inspector for the now defunct Inner London Education Authority, had interviewed a number of intelligent, articulate adults about their feelings about mathematics. He concluded that many, and in particular, those who had not been successful at mathematics in school, saw mathematics as:

1 Fixed, immutable, external, intractable, and uncreative.

2 Abstract and unrelated to reality.

3 A mystique accessible to a few.

4 A collection of rules and facts to be remembered.

5 An affront to common sense in some of the things it asserts.

6 A time-test.

7 An area in which judgements not only on one's intellect but on one's own personal worth will be made.

8 Concerned largely with computation.[1]

This is in stark contrast to the views of mathematicians, who frequently talk about the beauty of the subject, and use words like "elegant" to describe particular solutions to a problem. Buxton offered a different list, which much more closely accords with the views of those who are successful at learning mathematics:

1 Experimental, exploratory and creative.

2 Abstract at times but often related to the most practical of problems.

3 Open to all, but (as with all areas of study) to be penetrated more deeply by some than others.

4 A network of consistent relationships, easily remembered when understood.

5 Always reconcilable with the internal logic of the mind.

6 A contemplative subject requiring constant and undivided attention at times but almost never needing to be done in haste.

7 An area in which judgements on one's ability should carry no more weight than in other studies.

8 About relationships in general.[2]

The result is what Keith Stanovich has described as an educational "Matthew effect" based on the passage in the bible: to those that have, more shall be given and for those who have nothing, what little they have will be taken away.[3] Students who experience success in mathematics spend more and more time doing it, getting better and better, while those who find it more difficult come to believe that maths is not for them.

To break this vicious spiral, we have to make sure that students experience success in mathematics, because – as recent research has shown – in mathematics, success causes motivation at least as much as motivation causes success.[4] There is such a thing as 'talent' in mathematics to be sure – some students find learning mathematics easier than others – but as John Carroll showed over half a century ago, in school studies at least, talent, or 'aptitude', is really nothing more than an indication of how much time an individual will take to learn something.[5] Some students learn things quickly, and others take more time – and this is why opportunity cost is the single most important concept in improving educational achievement and closing achievement gaps. If some lessons are spent on things that are not necessary for the intended learning, then this will probably not have much impact on the achievement of students who find learning mathematics relatively easy. But for those who need more time, it is a disaster, and this is why "making every lesson count" is not just a good idea but a moral imperative. If we don't make every lesson count, then we widen the achievement gap. It is as simple as that.

In this book – the latest addition to the Making Every Lesson Count series – Emma McCrea applies the framework developed by Shaun Allison and Andy Tharby to the teaching of mathematics, and the result is a superb resource for anyone who teaches mathematics at any level. The book is well-written, concise – important when your message is that we should make every lesson count! – thoroughly grounded in the realities of teaching mathematics and authoritative, drawing out clear principles from the latest research on memory, learning and motivation. I could go on, but since opportunity cost is the most important concept in educational improvement, your time would be better spent reading what Emma has written …

Dylan Wiliam, UCL

Endnotes

1 Laurie Buxton, *Do You Panic About Maths? Coping with Maths Anxiety* (London: Heinemann Educational Books, 1981), p. 115.

2 Buxton, *Do You Panic About Maths?*, p. 116.

3 Keith E. Stanovich, Matthew Effects in Reading: Some Consequences of Individual Differences in the Acquisition of Literacy, *Reading Research Quarterly* 21(4) (1986): 360–407.

4 Gabrielle Garon-Carrier et al., Intrinsic Motivation and Achievement in Mathematics in Elementary School: A Longitudinal Investigation of Their Association, *Child Development* 87(1) (2016): 165–175. doi: 10.1111/cdev.12458

5 John B. Carroll, A Model for School Learning, *Teachers College Record* 64(8) (1963): 723–733.

Acknowledgements

It was the incredibly talented and charismatic author Peps Mccrea who started me on this journey when, in late 2017, he waved this tweet from Shaun Allison under my nose:

> Calling all #maths teachers – we are looking for an author to write 'Making Every Maths Lesson Count'. Interested?

What followed has been a remarkable adventure featuring bouts of pride (to hear that I was to be welcomed into the Making Every Lesson Count family), apprehension ("I do numbers, not words"), fear ("I'll never finish this") and excitement (at producing an actual book with my name on it).

I owe a great deal to those who helped along the way, particularly members of the incredible maths education community – a group of immensely talented super-humans who are constantly striving to help their students succeed. I hope I have done you justice.

To those who gave up their valuable time to review the book I thank you for your kind words and support. I am greatly indebted to Anne Watson, David Wees, Dylan Wiliam and Kris Boulton for their thoughtful feedback. Thanks to Dylan Wiliam for penning the thought-provoking foreword that sets the perfect tone for the book – it is a delight and an honour to share pages with such words of wisdom.

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Last, but not least, to M&M, who inspire me daily.

Contents

Introduction

Teaching maths is not easy. It can feel like all the cards are stacked against us. We teach a subject that is frequently loathed and publicly jibed. For many, "I was rubbish at maths" is worn as a badge of honour. The lack of choice that students get when learning maths can lead to a lack of buy-in. Yet at the same time it is regarded as one of the two most important subjects at GCSE in terms of accountability measures. These factors (and others) result in 40% of maths teachers leaving the profession within their first six years.[1] This retention crisis leaves our maths departments overstretched and our teachers overworked.

There is no doubt that learning maths and being numerate is important. There is a wealth of evidence about the personal and financial impact of poor numeracy to back this up. The Organisation for Economic Co-operation and Development (OECD) found that "high numeracy is particularly correlated with a higher likelihood of; having higher wages, having good to excellent health and being employed",[2] leading to the OECD's director for education and skills, Andreas Schleicher, to state that "good numeracy is the best protection against unemployment, low wages and poor health".[3]

A study by the Institute for Fiscal Studies calculated that students who are 'good' at maths are able to earn around an extra £2,100 per year,[4] and the government released a paper suggesting that getting five or more good GCSEs, including maths and English, would gain them an additional £100,000 over a lifetime.[5] Yet more than 20% of British adults have serious difficulties with numeracy that interfere with basic daily activities.[6] And the impact goes beyond the individual, with poor adult numeracy reportedly draining the UK economy to the tune of £20 billion per year.[7]

Yet there is a strong case that these reasons for learning maths are immaterial. Instead, we learn because we are human, because we are naturally inquisitive and possess an intrinsic thirst for knowledge. We learn because the more we know, the better able we are to make well-balanced, reasoned decisions. We learn maths because, in the words of the Department for Education, it "is a creative and highly interconnected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject."[8]

Our classrooms are incredibly complex places. At any given time, the factors influencing the effectiveness of our teaching and the ability of our students to learn are boundless – and this can lead to us relying on intuition as to what might work best. Yet as Yana Weinstein and Megan Sumeracki, co-founders of the Learning Scientists, point out, our own intuitions as to how we learn and how we should teach are not always correct.[9] This book tries to offer a route through this complexity. It does so by sharing a compendium of teaching strategies that are *evidence informed*; that is to say, the strategies presented draw from educational research and findings from the field of cognitive science.[10] Being evidence informed helps because it gives us a head start. Rather than using trial and error to identify what works in the classroom, which is a bit like trying to find a needle in a haystack, research evidence gives us a good starting point. Professor Daniel Muijs, head of research at Ofsted, goes one step further, suggesting that we need to be evidence informed because it is our moral duty (because it exists), for social justice (to close the gap) and for the credibility of our profession.[11]

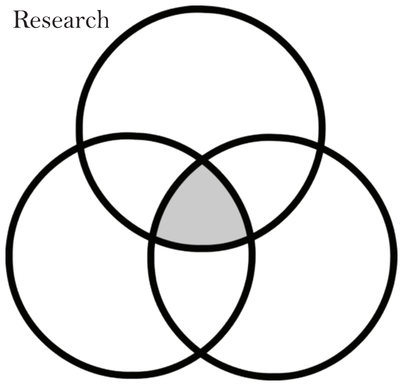


Figure I.1. The sweet spot is at the intersection of research, context and professional judgement

That is not to say that research outcomes will replicate as well, or in the same way, in every classroom. Being evidence informed is one part of the puzzle; adapting to context (type of school, ethos, local demographic, cohort, etc.) and using professional judgement (knowledge of students, values and beliefs) are others. A comparison can be drawn with building a skyscraper: there are principles of physics that need to be respected; principles that if they aren't followed would result in the building falling down. But these principles of physics don't tell you how to build the skyscraper or what it needs to look like.[12] As Dylan Wiliam eloquently said, "Everything works somewhere, and nothing works everywhere."[13]

Having an evidence-informed approach also helps us to avoid jumping on the fad bandwagon. For example, many years ago when I was head of maths, I dutifully 'tested' all the students to identify whether they were visual, auditory or kinaesthetic (VAK) learners and planned lessons accordingly, ensuring that each of these learning styles was catered for. We now know that this teaching strategy is a myth.[14]

The Six Principles

This book utilises the framework developed by Shaun Allison and Andy Tharby in *Making Every Lesson Count*.[15] The framework is comprised of six pedagogical principles that underpin great teaching and learning (see page 5).

The first principle is challenge – "the driving force of teaching".[16] In having high expectations of all our students, regardless of their prior attainment, we can help them to learn more, experience greater success and, most importantly, help them to foster higher expectations of themselves. In Chapter 1 we will also explore why getting students to attend to what it is that we want them to learn is so important.

What follows in the original *Making Every Lesson Count* are two chapters called 'Explanation' and 'Modelling'. This is where it gets a little tricky for us maths teachers. We rarely explain without a model due to the fact that our subject is best conveyed by modelling. Consequently, this book combines explanation and modelling (Chapter 2). We will also consider the role that cognitive load plays in learning and how we can use cognitive load theory to streamline our explanations.

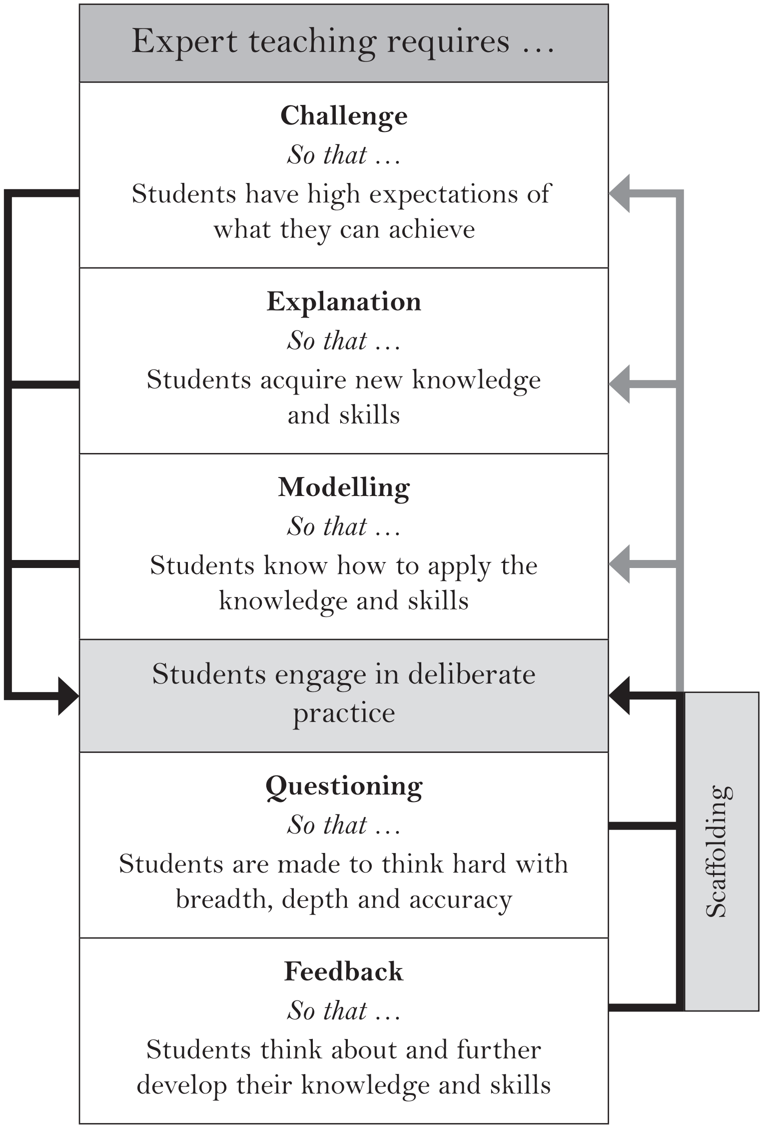
Naturally, practice comes next (Chapter 3), with a focus on how we can use the features of deliberate practice and embed retrieval practice strategies to supercharge the impact of the practice that students do.

We will then investigate how we can use effective questioning (Chapter 4) to either focus student attention or to gain an insight into their thinking. We will take a look at how effective questioning can help us overcome the 'curse of knowledge' and the Dunning–Kruger effect.

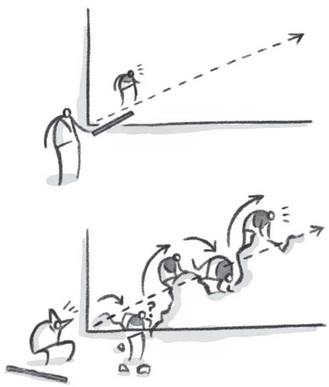
Last, but not least, is feedback (Chapter 5), the holy grail of teaching and learning principles. Just as the captain of a ship takes constant readings of weather and currents during a voyage, and then adjusts the speed and bearing in response to those readings, feedback allows teachers to do the same. It enables them to steer their students to their learning destination while making adjustments en route, both in the moment during the lesson and over time through changes to subsequent lessons and schemes of work. Without feedback, we are the captain of a ship adrift at sea.

While unpicking feedback, we also consider the implications of the idea that when we measure learning in a lesson we are not actually measuring learning – we are measuring performance. The measure of learning is if a student still knows and understands the content many months later.

Great maths teaching is aligned with all of these principles; however, they do not represent a lesson plan or a tick-list. This book presents them as individual entities, but in reality they are parts of a whole. They sustain each other. Not only do they help you to plan maths lessons and schemes of work, but they also help you to respond with spontaneity to the ever-changing and ever-complex needs of your students within lessons.


Expert teaching requires …
Challenge
So that …
Students have high expectations of what they can achieve
Explanation
So that …
Students acquire new knowledge and skills
Modelling
So that …
Students know how to apply the knowledge and skills
Students engage in deliberate practice
Questioning
So that …
Students are made to think hard with breadth, depth and accuracy
Feedback
So that …
Students think about and further develop their knowledge and skills
Scaffolding

In recent years, the education establishment has lionised the individual lesson. The unrelenting historic emphasis on lesson grading has led to an unhealthy focus on the success of single lessons (or lack thereof). The problem is that learning maths is not speedy, linear or logical. It is slow, erratic and messy, and does not readily conform to hour-long, bite-size chunks. In his book *The Hidden Lives of Learners*, Graham Nuthall brings together findings from various studies to uncover what really goes on in classrooms. One of the most surprising discoveries is that one in three things a student will have learned by the end of a lesson will not be known by any other student, despite our carefully planned and well-intended learning objectives.[17]



This book and the six principles behind it focus on what goes on in the classroom because that is where we are most able to influence change. Beyond our classrooms, there is a limit to the impact we can have – for example, we are unable to choose what mathematical content we feel is most appropriate for our students because this is mandated through the national curriculum.

The Sutton Trust, a foundation which improves social mobility in the UK through evidence-based programmes, research and policy advocacy, has found that the difference between a 'very effective' teacher and a 'poorly performing' teacher is considerable: "during one year with a very effective maths teacher, pupils gain 40% more in their learning than they would with a poorly performing maths teacher".[18] A different research project found that students in the most effective classrooms learn at four times the speed of those in the least effective classrooms.[19]

In *What Makes Great Teaching?*, Robert Coe and his colleagues from the Sutton Trust identify two features that show strong evidence of impact on student outcomes: pedagogical content knowledge and quality of instruction.[20] These features are within our influence (see Table I.1) and this book has a strong focus on both. However, deep subject knowledge and exceptional instructional design alone are not enough. Alongside these there is a need for excellent classroom management skills and a purposeful effort to foster good relationships based on mutual trust and respect.[21]

Table I.1. What are we able to influence in our teaching?

Within our influence	**Partially within our influence**	**Beyond our influence**
Instructional design Subject knowledge Relationships Classroom management Classroom learning climate	Departmental curriculum design Student attitude to maths Student attitude to learning Maths anxiety	Parental influence National curriculum content School learning climate Public attitude to maths Results accountability

One of the difficulties I faced when writing this book was the lack of clarity around the language of learning. For example, the term 'mastery' means different things to different teachers. If we don't have a common understanding of what something means, it can be very difficult to have a meaningful discussion about it. "'When *I* use a word,' Humpty Dumpty said, in rather a scornful tone, 'it means just what I choose it to mean – neither more nor less.'"[22]

To overcome this, I have tried to define exactly what I mean when I use a term. Table I.2 lists the words used throughout the book and their definitions that may otherwise be open to interpretation.

Table I.2. Terms used and their meaning

When I use the word …	**I am using it to mean …**
Example	The models we provide to students to aid their understanding.

When I use the word …	I am using it to mean …
Modelling	What we do as teachers to demonstrate a concept or approach (not modelling in the mathematical sense of the word).
Practice	What students do to improve their performance.
Problem	Any task presented to a student.
Problem solving[23]	Solving unfamiliar problems.
Question	A matter which we ask students to focus their attention on or use to elicit information from them.
Teaching	Our classroom practice – that is, what we as teachers do to help students learn.

We also need a shared understanding of what learning maths means. The American National Mathematics Advisory Panel suggests that learning maths requires three types of knowledge: factual, procedural and conceptual.[24] By *factual*, we mean retrieving from memory rather than calculating. For example, if I asked you to calculate 7 × 8 you would know the solution instantaneously because your response is automated. *Procedural* knowledge describes our ability to select and execute a procedure to solve a given problem – for example, to multiply 38 by 57 we might use the grid or column method. *Conceptual* knowledge, or conceptual understanding as it is more commonly called, relates to how well we understand mathematical concepts and the relationships between concepts. Using the previous example, this would mean having an understanding of what it means to multiply and why the grid or column methods (or even better, both) work.

There is an ongoing chicken and egg debate in maths about whether students should be taught *how* or *why* first. Some argue that if students have a conceptual understanding, they can create their own procedures; this is true for some but not all students. Others argue that conceptual understanding can be developed through procedural knowledge; however, there are students who know that to find the area of a rectangle they need to multiply the length by the width but cannot explain why. The evidence suggests that there is a bidirectional relationship between procedural knowledge and conceptual understanding, therefore they should be taught together wherever possible, so that one will reinforce the other.[25] Unfortunately conceptual understanding is the most difficult to acquire[26] and is developed over time.

It is when our students have secure factual and procedural knowledge *and* deep conceptual understanding that they have the best chance of success. Together, this knowledge enables them to build fluency. By 'fluency' we mean the ability to calculate accurately (find correct solutions), efficiently (using an appropriate strategy or algorithm with speed) and flexibly (adapting strategy and transferring across contexts).[27] It is this fluency that helps students to reason mathematically and solve unfamiliar problems.

We must not underestimate the importance of factual knowledge. As Willingham says, "Data from the last 30 years lead to a conclusion that is not scientifically challengeable: thinking well requires knowing facts … The very processes that teachers value the most – critical thinking processes like reasoning and problem solving – are intimately intertwined with factual knowledge that is in long-term memory."[28]

What is missing from this model is the barrier we face in terms of the attitudes and dispositions our students can sometimes display:

- *Poor motivation.* Attempts to motivate students before teaching them new material tend to be unsuccessful. It is better to allow them to experience success and then their motivation and confidence should follow. A good principle for this is that the best way to motivate students is through the content we teach them.
- *I'm rubbish at maths* is frequently heard. When a student says they are rubbish at maths, they are attributing their failure to succeed to something that they believe is unchangeable, which, in most cases, is simply not true.
- *When am I ever going to use this?* Possibly never, but the more they know about anything, the more they can understand about everything.
- *I can't do it.* Add the word *yet* and say: "You can't do it, *yet.* But with hard work and perseverance, you will be able to do it." This uses the language of growth mindset and reinforces the idea that nearly everyone is capable of learning maths.

Good relationships, built on mutual respect and trust, are vital when trying to foster positive attitudes and dispositions. Good relationships stem from having high expectations of students, both in their ability to learn and in their attitude and behaviour for learning, and being clear and consistent in your response when students do not meet these expectations.

Doug Lemov, the author of *Teach Like a Champion*, once quoted a friend as saying, "if you try to catch five rabbits, you catch none".[29] This struck me as a great metaphor for improving our teaching. If we try to add to or develop too many parts of our teaching at once, we are at risk of developing none. We see this in some schools where initiative after initiative is rolled out, never quite leaving enough time to embed the first one before the next one comes along. As you read this book, you will (hopefully) find several ideas that you will want to try in your teaching. Be sure to attempt them one at a time, using the reflective questions at the end of each chapter to support this.

Daniel Willingham asserts that "Teaching, like any complex cognitive skill, must be practiced to be improved."[30] For changes to our teaching to stick, we must make them habits. To form habits, we must practise regularly. Often we forget to do this, so setting ourselves reminders can help. For example, if you wanted to form the habit of going to bed a little earlier every evening to read a book, then setting an alarm on your phone might help until the habit is embedded. In a teaching context, perhaps you want to introduce the use of worked example pairs in your explanations (see Chapter 2). You could add them to your lesson plan pro forma, write a reminder on a sticky note to go on your teacher planner or PC screen or stick a big sign on your desk. Alternatively, tell a colleague about your intentions. We stand a greater chance of success when we share our goals.

Finally, in the words of John Hattie, the author of *Visible Learning*, "know thy impact".[31] We need to be able to measure the impact of the changes we make to our teaching. When I use the word 'measure', I use it loosely. With regard to the example of using worked example pairs during explanations, we could measure impact by reflecting on whether the students are better able to get started on their independent practice without further teacher support. Always try to find a way to assess impact, even if it is a crude measure. Ask yourself, "What change do I expect to see?" As Robert Coe observes, "Whenever we make a change we must try to evaluate its impact as robustly as we can."[32]

In summary, as you read this book, it might be useful to make a list of what you would like to try in your classroom. Select one strategy which you feel will make a powerful change to your teaching, try to work out a way to measure the impact of this change, use reminders to form a habit so that it becomes embedded in your teaching, and then take a moment to

Writing in the practical, engaging style of the award-winning *Making Every Lesson Count*, experienced maths teacher and lecturer Emma McCrea takes away the guesswork as she sums up the key components of effective maths teaching.

Making Every Maths Lesson Count is underpinned by six pedagogical principles – challenge, explanation, modelling, practice, feedback and questioning – and shares 52 high-impact strategies designed to streamline teacher workload and ramp up the level of challenge in the classroom.

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Jo Morgan, maths teacher and creator of resourceaholic.com

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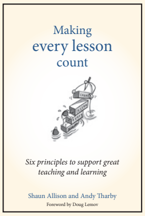
Anne Watson, Professor Emerita of Mathematics Education, University of Oxford



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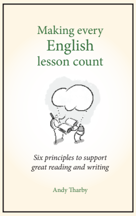

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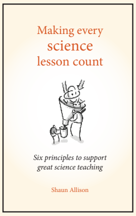

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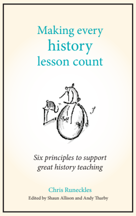

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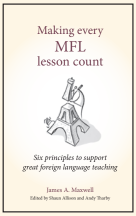

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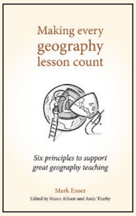

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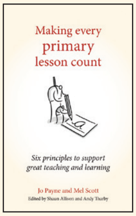

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